Algebra II
Unit 1: Functions and Inverse Functions

Pacing: 4 weeks + 1 week for reteaching/enrichment

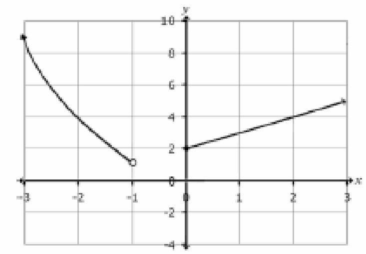
Mathematical Practices
Mathematical Practices #1 and #3 describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. *Practices in bold are to be emphasized in the unit.* **1. Make sense of problems and persevere in solving them.** 2. Reason abstractly and quantitatively. **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics.** 5. Use appropriate tools strategically. 6. Attend to precision. **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**
Standards Overview
Analyze functions using different representations. **Build a function that models a relationship between two quantities.** **Build new functions from existing functions.**

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

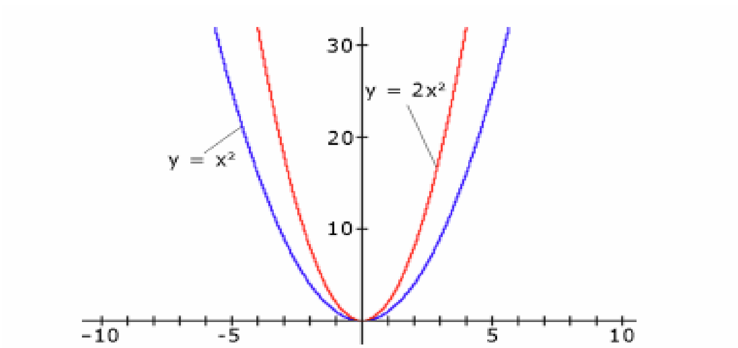
Algebra II
Unit 1: Functions and Inverse Functions

<table>
<tr><th>Priority and Supporting CCSS</th><th>Explanations and Examples*</th></tr>
<tr><td>CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*</td><td rowspan="2">Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.

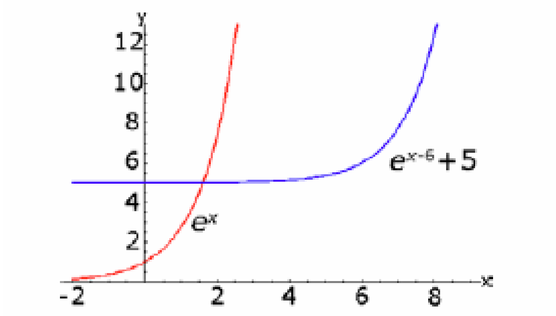
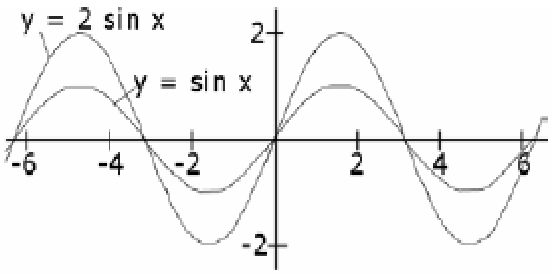
Examples:
• Describe key characteristics of the graph of $f(x) = |x - 3| + 5$.

• Sketch the graph and identify the key characteristics of the function described below.
$F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$

• Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph.
• Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes.
• Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?</td></tr>
<tr><td>CC.9-12.F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</td></tr>
<tr><td>CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td><td></td></tr>
</table>

Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

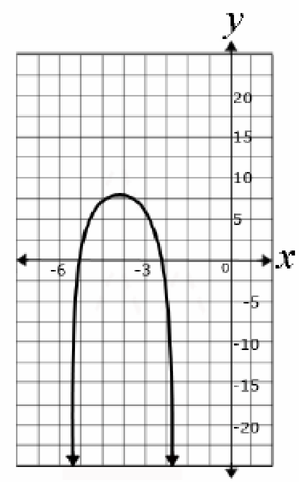
Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions  • Describe effect of varying the parameters *a, h,* and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$. • Compare the shape and position of the graphs of $f(x) = e^x$ to $g(x) = e^{x-6} + 5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
	 • Describe the effect of varying the parameters *a, h,* and *k* on the shape and position of the graph f(*x*) = $ab^{(x+h)}$ + k., orally or in written format. What effect do values between 0 and 1 have? What effect do negative values have? • Compare the shape and position of the graphs of *y* = sin *x* to *y* = 2 sin *x.* 

Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 1: Functions and Inverse Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*.*	Students may explain orally, or in written format, the existing relationships.
CC.9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	Example: ● Examine the functions below. Which function has the larger maximum? How do you know? $f(x) = -2x^2 - 8x + 20$ 

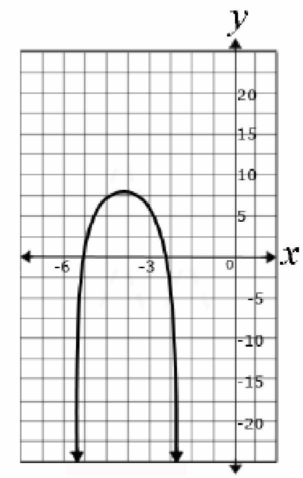
Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples: ● For the function $h(x) = (x - 2)^3$, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist. ● Graph $h(x)$ and $h^{-1}(x)$ and explain how they relate to each other graphically. ● Find a domain for $f(x) = 3x^2 + 12x - 8$ on which it has an inverse. Explain why it is necessary to restrict the domain of the function.
CC.9-12.F.BF.1 Write a function that describes a relationship between two quantities.*	Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples: ● You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. ● A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F
CC.9-12.F.BF.1c (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.	

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
	decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. ● The radius of a circular oil slick after t hours is given in feet by $r = 10t^2 - 0.5t$, for $0 \leq t \leq 10$. Find the area of the oil slick as a function of time.
CC.9-12.F.BF.4b (+) Verify by composition that one function is the inverse of another.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples: ● For the function $h(x) = (x - 2)^3$, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist. ● Graph $h(x)$ and $h^{-1}(x)$ and explain how they relate to each other graphically. ● Find a domain for $f(x) = 3x^2 + 12x - 8$ on which it has an inverse. Explain why it is necessary to restrict the domain of the function.
CC.9-12.F.BF.4 Find inverse functions	
CC.9-12.F.BF.4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ for $x > 0$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$ (x not equal to 1).	
CC.9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	Examples: • $\sqrt{x+2} = 5$ • $\frac{7}{8}\sqrt{2x-5} = 21$ • $\frac{x+2}{x+3} = 2$Solv • $\sqrt{3x-7} = -4$

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	Example: ● Examine the functions below. Which function has the larger maximum? How do you know? $f(x) = -2x^2 - 8x + 20$ 
CC.9-12.F.BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.BF.4d (+) Produce an invertible function from a non-invertible function by restricting the domain.	Examples: ● For the function $h(x) = (x - 2)^3$, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist. ● Graph $h(x)$ and $h^{-1}(x)$ and explain how they relate to each other graphically. ● Find a domain for $f(x) = 3x^2 + 12x - 8$ on which it has an inverse. Explain why it is necessary to restrict the domain of the function.

Algebra II
Unit 1: Functions and Inverse Functions

Concepts What Students Need to Know	Skills What Students Need To Be Able To Do	Bloom's Taxonomy Levels
● Functions (expressed symbolically) o Square root o Cube root o Piecewise-defined (includes step and absolute value) ● Key Features o Intercepts o intervals ➢ increasing or decreasing, ➢ positive or negative o relative maximums and minimums o symmetries o end behavior / endpoints ● Technology (graphing complicated functions)	● Graph ● Show key features ● Use technology	3 4 3
● Functions	● Write a function ● Compose functions & understand in terms of context of the problem	3 4

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 1: Functions and Inverse Functions

Concepts What Students Need to Know	Skills What Students Need To Be Able To Do	Bloom's Taxonomy Levels
• Inverse functions	• Find inverse functions and attend to domain e.g. restrictions	4
• Equation (of form f(x)=c)	• Solve equation	3
	• Write expression	3

Essential Questions
How can the relationship between quantities best be represented?
Corresponding Big Ideas
Equations, verbal descriptions, graphs, and tables provide insight into the relationship between quantities.
Standardized Assessment Correlations (State, College and Career)
CollegeBoard PSAT and SAT

Unit Vocabulary
Function, domain, range, x and y-intercepts, symmetry, even and odd functions, intervals of increase and decrease, local maximum, local minimum, quadratic, cubic, square root, cube root, rational function, piece-wise, constant function, greatest integer function, transformations (vertical and horizontal shifts, vertical compression and stretch, horizontal compression and stretch, reflection about the x-axis, reflection about the y-axis), composition, domain of composite functions, inverse functions, symmetry of inverse functions, domain of inverse functions

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 1: Functions and Inverse Functions

Unit Assessments **The items developed for this section can be used during the course of instruction when deemed appropriate by the teacher.**
Section quizzes End-of-unit test

Learning Activities

Topic	Section in Text	CCSS
Reading a graph • Given input, find output • Given output, find input • Identify domain and range	Pre Calc 2.2	F.IF.7 F.IF.9
Function notation - algebraically • Given input, find output • Given output, find input • Composition of functions – given one input, find output of composition	PH Algebra 2 2.1 7.6 Pre-Calc 2.1 5.1	F.IF.7 F.BF.1 F.BF.1c(+) A.REI.2
Algebraically finding features of functions • Input and output • x and y-intercepts • Applications – explain what input and output mean in context to problem	Pre Calc 2.2	F.IF.7 A.REI.2

More features of functions • Domain and range • Intervals of increase and decrease • Local maximum and/or minimum • Where is a function constant • Even vs. Odd (graphically) • Applications – finding a maximum or minimum and explain within context to problem	Pre Calc 2.3	F.IF.5 F.IF.7
Parent Functions • know quadratic, square root, cubic, cube root, rational • transformations of these functions • work with piece-wise functions	Pre-Calc 2.4 2.5 PH Algebra 2 (vary scattered throughout text) p. 71 (extension) 2.5 2.6 5.3 7.8 8.2 8.3 9.2 9.3	F.BF.3 F.IF.7b A.CED.2
Inverse Functions • Finding the inverse a function both graphically and algebraically	Pre-Calc 5.2 PH Algebra 2	F.BF.4 F.BF.4a

• Verify by composition one function is the inverse of another	7.7 Activity Lab p. 413 8.3	F.BF.4b(+) F.BF.4c(+) F.BF.4d(+) F.BF.1 F.BF.1c(+)

Application
1. At time 0 seconds an elevator starts 400 feet above the ground. After 8 seconds the elevator is 320 feet above the ground. 2. A squirrel sitting in a tree drops an acorn. The function $h(t) = -16t^2 + 24$ gives the height of the acorn h in feet after t seconds. (a) Identify the independent and dependent variables. (b) Using function notation, evaluate the function for each given value of t. Explain the meaning of your answer in the context of the situation. $t = 0$, $t = 0.5$ (c) Is 2 a reasonable domain value? Explain why or why not. (d) Determine when the acorn reaches 12 feet.

Algebra II
Unit 2: Polynomial Functions

Pacing: 5 weeks + 1 week for reteaching/enrichment

Mathematical Practices
Mathematical Practices #1 and #3 describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. *Practices in bold are to be emphasized in the unit.* **1. Make sense of problems and persevere in solving them.** **2. Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. **6. Attend to precision.** **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**
Standards Overview
Use complex numbers in polynomial identities and equations. **Create equations that describe numbers or relationships.** **Analyze functions using different representations.**

Algebra II
Unit 2: Polynomial Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.	Examples: ● Within which number system can *x*2 = – 2 be solved? Explain how you know. ● Solve *x*2+ 2*x* + 2 = 0 over the complex numbers. ● Find all solutions of $2x^2 + 5 = 2x$ and express them in the form *a* + *bi.*
CC.9-12.N.CN.1 Know there is a complex number *i* such that $i^2 = \sqrt{-1}$, and every complex number has the form *a* + *bi* with *a* and *b* real.	
CC.9-12.N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers	Example: ● Simplify the following expression. Justify each step using the commutative, associative and distributive properties. $(3 - 2i)(-7 + 4i)$ Solutions may vary; one solution follows: $(3 - 2i)(-7 + 4i)$ $3(-7 + 4i) - 2i(-7 + 4i)$ Distributive Property $-21 + 12i + 14i - 8i^2$ Distributive Property $-21 + (12i + 14i) - 8i^2$ Associative Property $-21 + i(12 + 14) - 8i^2$ Distributive Property $-21 + 26i - 8i^2$ Computation $-21 + 26i - 8(-1)$ $i^2 = -1$

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 2: Polynomial Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
	$-21 + 26i + 8$ Computation $-21 + 8 + 26i$ Commutative Property $-13 + 26i$ Computation
CC.9-12.N.CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.	
CC.9-12.N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials	Examples: ● How many zeros does $-2x^2 + 3x – 8$ have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra. ● How many complex zeros does the following polynomial have? How do you know? $p(x) = (x^2 -3) (x^2 +2)(x - 3)(2x – 1)$
CC.9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b	Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.

<table>
<tr><th>Priority and Supporting CCSS</th><th>Explanations and Examples*</th></tr>
<tr><td></td><td>

Value of Discriminant	Nature of Roots	Nature of Graph
$b^2 - 4ac = 0$	1 real roots	intersects x-axis once
$b^2 - 4ac > 0$	2 real roots	intersects x-axis twice
$b^2 - 4ac < 0$	2 complex roots	does not intersect x-axis

- Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation.
- What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?

</td></tr>
<tr><td>

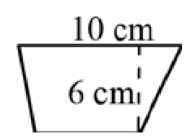
CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

</td><td>

Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.

Examples:

- Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.



Lava coming from the eruption of a volcano follows a parabolic path. The height *h* in feet of a piece of lava *t* seconds after it is ejected from the volcano is given by $h(t) = -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet?

</td></tr>
</table>

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

<table>
<tr><th>Priority and Supporting CCSS</th><th>Explanations and Examples*</th></tr>
<tr><td>CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td><td></td></tr>
<tr><td>CC.9-12.A.REI.4 Solve quadratic equations in one variable

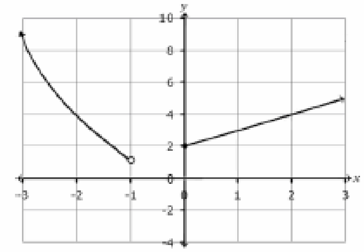
CC.9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b</td><td>Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.

<table><tr><th>Value of Discriminant</th><th>Nature of Roots</th><th>Nature of Graph</th></tr><tr><td>$b^2 – 4ac = 0$</td><td>1 real roots</td><td>intersects x-axis once</td></tr><tr><td>$b^2 – 4ac > 0$</td><td>2 real roots</td><td>intersects x-axis twice</td></tr><tr><td>$b^2 – 4ac < 0$</td><td>2 complex roots</td><td>does not intersect x-axis</td></tr></table>
● Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation.

● What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?</td></tr>
<tr><td>CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*</td><td>Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.continued on next page</td></tr>
</table>

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 2: Polynomial Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior	Examples: • Describe key characteristics of the graph of $f(x) = \lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x) = \begin{cases} x+2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. • Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?
CC.9-12.A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further. Example: Factor $x^3 - 2x^2 - 35x$

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 2: Polynomial Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	
CC.9-12.A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	The Remainder theorem says that if a polynomial $p(x)$ is divided by $x – a$, then the remainder is the constant $p(a)$. That is, $p(x)=q(x)(x-a)+p(a)$. So if p(a) = 0 then p(x) = q(x)(x-a). ● Let $p(x)=x^5 -3x^4 +8x^2 -9x+30$. Evaluate p(-2). What does your answer tell you about the factors of p(x)? [Answer: p(-2) = 0 so x+2 is a factor.]
CC.9-12.A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial	Graphing calculators or programs can be used to generate graphs of polynomial functions. Example: Factor the expression $x^3 + 4x^2 –59x –126$ and explain how your answer can be used to solve the equation $x^3 + 4x^2 –59x –126 = 0$. Explain why the solutions to this equation are the same as the x-intercepts of the graph of the function $f(x) = x^3 + 4x^2 –59x =126$.
CC.9-12.A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples	Examples: Use the distributive law to explain why $x^2 – y^2 = (x – y)(x + y)$ for any two numbers x and y. Derive the identity $(x – y)^2 = x^2 – 2xy + y^2$ from $(x + y)^2 = x^2 + 2xy + y^2$ by replacing y by $–y$. Use an identity to explain the pattern

Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 2: Polynomial Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
	continued on next page $2^2 – 1^2 = 3$ $3^2 – 2^2 = 5$ $4^2 – 3^2 = 7$ $5^2 – 4^2 = 9$ [Answer: $(n + 1)^2 - n^2 = 2n + 1$ for any whole number n.]
CC.9-12.A.APR.5 (+) Know and apply that the Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)	Examples: • Use Pascal's Triangle to expand the expression $(2x-1)^4$. • Find the middle term in the expansion of $(x^2+2)^{18}$. • 1 1 1 1 2 1 $(x+1)^3 = x^3+3x^2+3x+1$ 1 3 3 1 ← 1 4 6 4 1 ↑ ↑ ↑ ↑ ↑ $_4C_0$ $_4C_1$ $_4C_2$ $_4C_3$ $_4C_4$
CC.9-12.N.CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	Examples: ● How many zeros does $–2x^2 +3x –8$ have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra. ● How many complex zeros does the following polynomial have? How do you know?

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
	$p(x)=(x^2-3)(x^2+2)(x-3)(2x-1)$
CC.9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. Examples: ● A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet. o What is a reasonable domain restriction for t in this context? o Determine the height of the rocket two seconds after it was launched. o Determine the maximum height obtained by the rocket. o Determine the time when the rocket is 100 feet above the ground. o Determine the time at which the rocket hits the ground. o How would you refine your answer to the first question based on your response to the second and fifth questions?

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
	• Compare the graphs of $y = 3x^2$ and $y = 3x^3$. • Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$. • Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease. • It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.
CC.9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	Example: Contrast the growth of the $f(x)=x^3$ and $f(x)=3^x$.

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 2: Polynomial Functions

Concepts What Students Need to Know	Skills What Students Need To Be Able To Do	Bloom's Taxonomy Levels
● Quadratic equations	● Solve quadratic equations	3
o Real coefficients		
o Complex solutions		
● Quadratic Equations and Inequalities	● Create equations	3
● Real world problem	● Solve real world problems	4
● Functions (expressed symbolically)	● Graph polynomial functions	3
o Polynomial functions	● Identify zeros	1
o Zeros of polynomials	● Show key features & end behavior	4
o Suitable factorizations	● Use technology	3
● Key Features		
o Intercepts		
o intervals		
➢ increasing or decreasing,		
➢ positive or negative;		
o relative maximums and minimums		

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Concepts What Students Need to Know	Skills What Students Need To Be Able To Do	Bloom's Taxonomy Levels
o symmetries o end behavior ● Technology (graphing complicated functions)		

Algebra II
Unit 2: Polynomial Functions

Essential Questions
In what ways can functions be built? How does knowledge of real numbers help when working with complex numbers? How can the relationship between quantities best be represented?
Corresponding Big Ideas
Functions can be created by identifying the pattern of a relationship or by applying geometric transformations to an existing function.
Standardized Assessment Correlations (State, College and Career)
CollegeBoard PSAT and SAT

Unit Vocabulary
Polynomials, Quadratic, Cubic, Quartic, Quintic, degree of polynomial, square root method, completing the square, quadratic formula, factoring, definition of "I"complex solutions (a + bi), zeros, roots, intercepts, intervals of increase and decrease, relative maximums and minimums, multiplicity of zeros, symmetry, end behaviors, Fundamental Theorem of Algebra, Remainder Theorem, Rational Root Theorem, Binomial Theorem and Pascal's Triangle

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 2: Polynomial Functions

Unit Assessments **The items developed for this section can be used during the course of instruction when deemed appropriate by the teacher.**
Section quizzes End-of-unit test

Learning Activities

Topic	Section in Text	CCSS
Operate on Polynomials- add, subtract and multiply • (+)Know and apply the Binomial Theorem, use Pascal's Triangle	PH Alg 2 6.1 6.8	A.APR.1 A.APR.5
Complex Numbers • Know $i^2 = -1$ • Know form a + bi • Add, Subtract and multiply with the set of complex numbers	5.6 PC A.7 (appendix)	N.CN.1 N.CN.2
Solve Quadratic Equations that have complex solutions (a + bi) • Use the Square Root Method • Use the Zero-Product Property • Solve by Factoring (+)including x^2+4	5.4 5.5 5.7 5.8	A.REI.4a A.REI.4b N.CN.7 N.CN.8

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

• Solve by Completing the Square • Solve using the Quadratic Formula • Choose appropriate method of solving based on the initial form of the equation	PC A.7	
Use factoring skills to factor higher degree polynomials • Sum and Difference of Two Squares – higher than a quadratic degree ($x^4 - y^4$) • Sum and Difference of Two Cubes • Quartic Trinomials fitting the quadratic pattern • Grouping	6.4 6.2 6.3 PC 4.5 4.6	A.SSE.2
Graph polynomial functions • Show key features of graph: zeros, end behavior, y-intercept, intervals of increase and decrease, relative maximums and minimums, symmetry • Simple functions by hand • Use technology for more complicated	Extension p. 312 PC 4.1	F.IF.4 F.IF.7c A.APR.3
Create equations and inequalities: Linear, Quadratic and Polynomial • Solve using appropriate method • Graph the equations	6.2 6.4 6.5	A.CED.1 A.CED.2

Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

	PC 4.1 4.4	
(+)Fundamental Theorem of Algebra	6.6 PC 4.5 4.6	N.CN.9
Know and Apply the Remainder Theorem	6.3 6.5 6.6 PC 4.5	A.APR.2
(+)Prove polynomial identities		A.APR.4

Application

1. Given: $g(x) = -0.2(x-4)^2(x+1)(x+3)$

    a. State the degree *f(x).* Describe the end behaviors of the function.
    b. Determine the zeros. State any multiplicity and if each would touch or cross the x-axis.
    c. Find the y-intercept.
    d. Using the graphing calculator, find the ordered pairs that represent the relative maximum and minimum.
    e. Sketch *f(x)*. Labeling all key points mentioned above.
    f. Find the Domain and Range
    g. Find the interval(s) of increase and decrease
    h. Solve f(x) > 0. Use interval notation.

2. A block of cheese is a cube whose side is x in. long. You cut off a 1-inch thick piece from the right side. Then you cut off a 3-inch thick piece from the front. The slices are perpendicular to each other. The volume of the remaining block is 2002 in^3. What are the dimensions of the original block of cheese?

Algebra II
Unit 3: Rational Expressions and Functions

Pacing: 3 weeks + 1 week for reteaching/enrichment

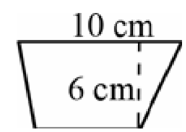
Mathematical Practices
Mathematical Practices #1 and #3 describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. *Practices in bold are to be emphasized in the unit.* 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. **4. Model with mathematics.** **5. Use appropriate tools strategically.** 6. Attend to precision. **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**
Standards Overview
Understand solving equations as a process of reasoning and explain the reasoning. **Analyze functions using different representations.**

Algebra II
Unit 3: Rational Expressions and Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise	Examples: • $\sqrt{x+2}=5$ • $\frac{7}{8}\sqrt{2x-5}=21$ • $\frac{x+2}{x+3}=2$Solv • $\sqrt{3x-7}=-4$
CC.9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*	Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret there meaning in terms of a context.
CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P	
CC.9-12.A.APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system	The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. Examples: • Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. • Express $f(x)=\frac{2x+1}{x-1}$ in a form that reveals the horizontal asymptote of its graph. [Answer: $f(x)=\frac{2x+1}{x-1}=\frac{2(x-1)+2}{x-1}=2+\frac{2}{x-1}$, so the horizontal asymptote is $y = 2$.]

Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 3: Rational Expressions and Functions

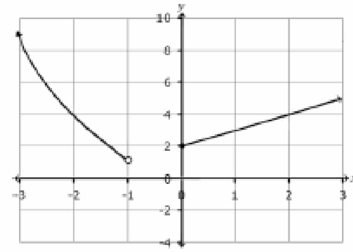
Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions	Examples: • Use the formula for the sum of two fractions to explain why the sum of two rational expressions is another rational expression. • Express $\frac{1}{x^2+1} - \frac{1}{x^2-1}$ in the form $a(x)/b(x)$, where $a(x)$ and $b(x)$ are polynomials.
CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. Examples: • Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.  Lava coming from the eruption of a volcano follows a parabolic path. The height *h* in feet of a piece of lava *t* seconds after it is ejected from the volcano is given by $h(t)= -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet?
CC.9-12.A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	Example: • A club is selling hats and jackets as a fundraiser. Their budget is \$1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs \$5 and each jacket costs \$8.

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
	o Write a system of inequalities to represent the situation. o Graph the inequalities. o If the club buys 150 hats and 100 jackets, will the conditions be satisfied? o What is the maximum number of jackets they can buy and still meet the conditions?
CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.
CC.9-12.F.IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior	

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 3: Rational Expressions and Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
	Examples: • Describe key characteristics of the graph of *f(x)* = $\lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. • Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?

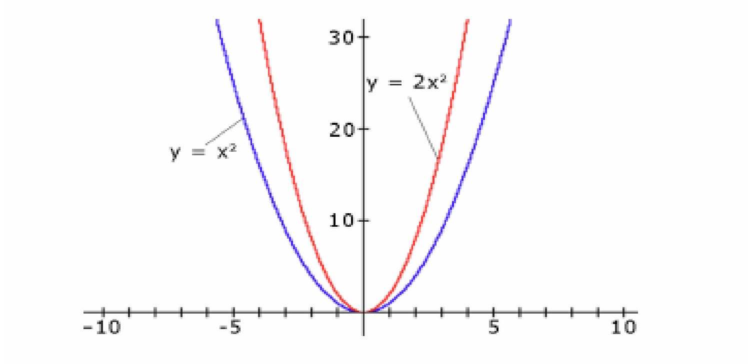
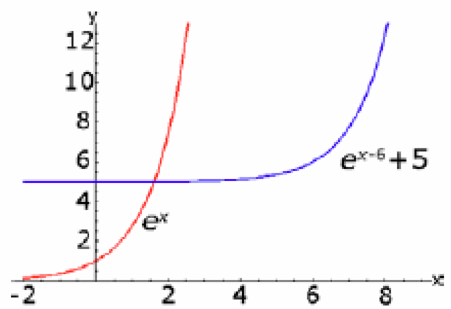
Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 3: Rational Expressions and Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions Example: ● Given the following equations determine the *x* value that results in an equal output for both functions. $f(x) = 3x – 2$ $g(x) = (x + 3)^2 – 1$
CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions

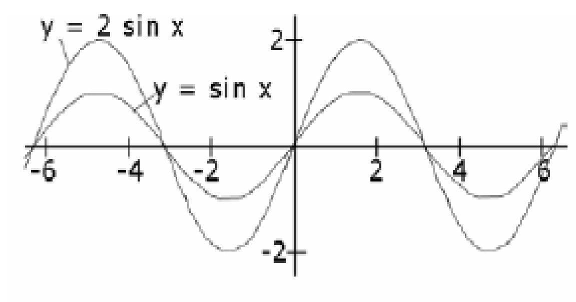
Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 3: Rational Expressions and Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
	 • Describe effect of varying the parameters *a, h,* and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$. • Compare the shape and position of the graphs of $f(x) = e^x$ to $g(x) = e^{x-6} + 5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions 

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 3: Rational Expressions and Functions

Priority and Supporting **CCSS**	**Explanations and Examples***
	• Describe the effect of varying the parameters *a, h,* and *k* on the shape and position of the graph f(*x*) = $ab^{(x + h)}$ + k., orally or in written format. What effect do values between 0 and 1 have? What effect do negative values have? • Compare the shape and position of the graphs of *y* = sin *x* to *y* = 2 sin *x.* 

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 3: Rational Expressions and Functions

Concepts What Students Need to Know	Skills What Students Need To Be Able To Do	Bloom's Taxonomy Levels
● Simple rational equations (leads to linear or quadratic equation)	● Solve	3
● Extraneous solution examples	● Give examples	4
● Functions (expressed symbolically)	● Graph	3
o Related to $y = 1/x$	● Show key features & end behavior	4
o Related to $y = 1/x^2$	● Use technology	3
● Key Features	● Identify (zeros, asymptotes)	2
o Intercepts		
o intervals		
➢ increasing or decreasing		
➢ positive or negative		
o relative maximums and minimums		
o symmetries		
o end behavior		
● Technology (graphing complicated functions)		

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 3: Rational Expressions and Functions

Essential Questions
How can the properties of the real number system be useful when working with polynomials and rational expressions?
Corresponding Big Ideas
Algebraic expressions, such as polynomials and rational expressions symbolize numerical relationships and can be manipulated in much the same way as numbers.
Standardized Assessment Correlations (State, College and Career)
CollegeBoard PSAT and SAT

Unit Vocabulary
Inverse, joint and combined variation, reciprocal function, rational function, branch, symmetry, restriction/point of discontinuity, vertical, horizontal and oblique asymptote, end behavior, hole, relative maximum or minimum, intervals of increase and decrease, domain and range, simplest form of a rational expression, complex fraction, transformations, solutions to a system

Unit Assessments The items developed for this section can be used during the course of instruction when deemed appropriate by the teacher.
Section quizzes, End-of-unit test

Algebra II
Unit 3: Rational Expressions and Functions

Learning Activities

Topic	Section in Text	CCSS
Graph Reciprocal Function Family • Parent Function f(x) = 1/x • Symmetry of • Transformations of the parent function • Find zeros and asymptotes, show end behavior • Write an equation involving a reciprocal function	PH Algebra 2 9.2 Pre-Calc 4.2	F.IF.7 F.IF.7d F.BF.3 A.CED.1
Inverse Variation	9.1	A.SSE.1 A.SSE.1b A.CED.1
Rational Function Graphs • Define rational function as quotient of two polynomial functions • Find zeros, y-intercept, asymptotes(vertical, horizontal, oblique), holes and understand end behavior • Rewrite a(x)/b(x) in the form of q(x) + r(x)/b(x) using long division	Activity Lab p. 494 9.3 Pre-Calc 4.2 4.3	A.APR.6 F.IF.7 F.IF.7d A.CED.1 A.CED.3

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 3: Rational Expressions and Functions

• Graph a rational function • Relative maximum or minimum • Interval of increase or decrease • Model problems with a rational function and understand constraints (points of discontinuity)		
Operate with rational expressions • Identify restrictions/points of discontinuity • Simplify an expression by reducing common factors • Perform multiplication, division, addition and subtraction	9.4 9.5 Pre-Calc Appendix A.5 p. A35	A.APR.7
Solving Rational Equations • Find Zeros algebraically • Use Graphing Calculator to find zeros • Model problem with rational equation and solve • Identify and interpret "*no solution*"	9.6 Pre-Calc Appendix A.6 p. A46	A.REI.2 A.CED.1 A.CED.3
Solve a system of Equations including Rational Equations • Graphically • Using technology • Algebraically	9.6 Teacher supplemental materials	A.REI.11

Application
1. Holt can power wash a house in 6 hours. Chad con power wash a similar house in 7 hours. How long will it take them if they work together? 2. The function $C(t) = \frac{5t}{0.01t^2 + 3.3}$ describes the concentration of a drug in the blood system over time. In this case, the medication was taken orally. *C* is measured in micrograms per milliliter and *t* is measured in minutes. a.) Sketch a graph of the function over the first two hours after the dose is given. Label the axes. b.) Determine when the maximum amount of the drug is in the body and the amount at that time. c.) Explain within the context of the problem the shape of the graph between takin the medication orally (t = 0) and the maximum point. What does the shape of the graph communicate between the maximum point and two hours after taking the drug? d.) What are the asymptotes of the given function? What is the meaning of the asymptotes within the context of the problem?

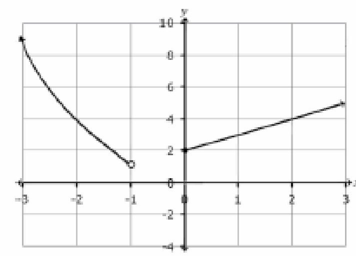
Algebra II
Unit 4: Trigonometric Functions

Pacing: 4 weeks + 1 week for reteaching/enrichment

Mathematical Practices
Mathematical Practices #1 and #3 describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. *Practices in bold are to be emphasized in the unit.* 1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics.** 5. Use appropriate tools strategically. **6. Attend to precision.** **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**
Standards Overview
Analyze functions using different representations. **Extend the domain of trigonometric functions using the unit circle.** **Model periodic phenomena with trigonometric functions.**

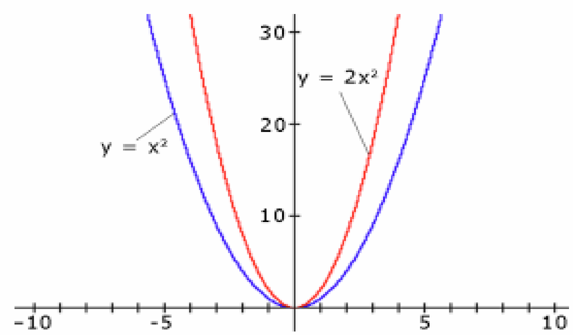
Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 4: Trigonometric Functions

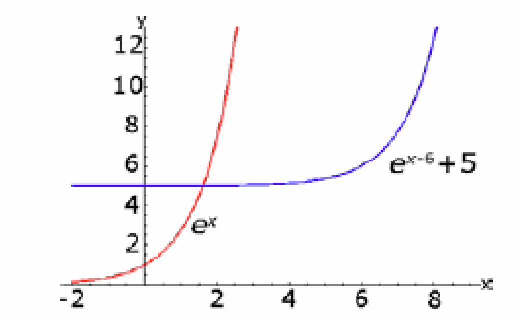
Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.
CC.9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude	Examples: • Describe key characteristics of the graph of $f(x) = \lvert x - 3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x) = \begin{cases} x + 2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. • Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

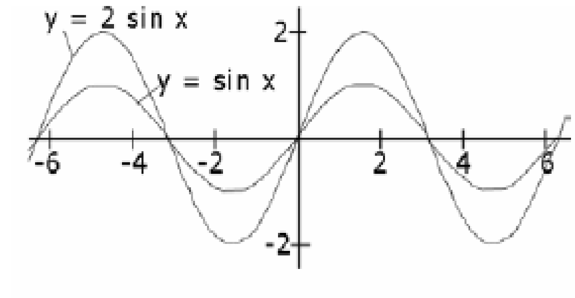
Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.1F.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. Examples: ● A rocket is launched from 180 feet above the ground at time t = 0. The function that models this situation is given by $h = -16t2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet. o What is a reasonable domain restriction for t in this context? o Determine the height of the rocket two seconds after it was launched. o Determine the maximum height obtained by the rocket. o Determine the time when the rocket is 100 feet above the ground. o Determine the time at which the rocket hits the ground. o How would you refine your answer to the first question based on your response to the second and fifth questions? ● Compare the graphs of $y = 3x^2$ and $y = 3x^3$. ● Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$. ● Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end

Priority and Supporting **CCSS**	**Explanations and Examples***
	behavior and any intervals of constancy, increase, and decrease. • It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.
CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions 

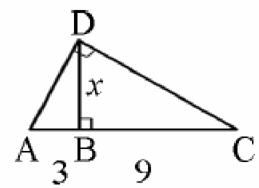
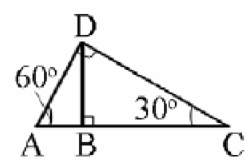
Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
	• Describe effect of varying the parameters *a, h,* and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$. • Compare the shape and position of the graphs of $f(x) = e^x$ to $g(x) = e^{x-6} + 5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions  • Describe the effect of varying the parameters *a, h,* and *k* on the shape and position of the graph $f(x) = ab^{(x + h)} + k$., orally or in written format. What effect do values between 0 and 1 have? What effect do negative values have? • Compare the shape and position of the graphs of $y = \sin x$ to $y = 2 \sin x$.

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
	
CC.9-12.F.TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or written format) their understanding of symmetry and periodicity of trigonometric functions.
CC.9-12.F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or in written format) their understanding.
CC.9-12.F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number	Examples: • Evaluate all six trigonometric functions of $\theta = \frac{\pi}{3}$. • Evaluate all six trigonometric functions of $\theta = 225°$. • Find the value of x in the given triangle where $\overline{AD} \perp \overline{DC}$ and $\overline{AC} \perp \overline{DB}$ $m\angle A = 60°, m\angle C = 30°$. Explain your process for solving the problem including the use of trigonometric ratios as appropriate.  • Find the measure of the missing segment in the given triangle where $\overline{AD} \perp \overline{DC}$, $\overline{AC} \perp \overline{DB}$, $m\angle A = 60°, m\angle C = 30°, \overline{AC} = 12, \overline{AB} = 3$. Explain (orally or in written format) your process for solving the problem including use of trigonometric ratios as appropriate. 
CC.9-12.F.TF.8 Prove the Pythagorean identity $(\sin A)^2 + (\cos A)^2 = 1$ and use it to calculate trigonometric ratios.	

Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions and periodic phenomena. Example: ● The temperature of a chemical reaction oscillates between a low of 20°C and a high of 120°C. The temperature is at its lowest point when *t* = 0 and completes one cycle over a six hour period. a) Sketch the temperature, *T*, against the elapsed time, *t*, over a 12 hour period. b) Find the period, amplitude, and the midline of the graph you drew in part a). c) Write a function to represent the relationship between time and temperature. d) What will the temperature of the reaction be 14 hours after it began? e) At what point during a 24 hour day will the reaction have a temperature of 60° C?
CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales	

Algebra II
Unit 4: Trigonometric Functions

Concepts What Students Need to Know	Skills What Students Need To Be Able To Do	Bloom's Taxonomy Levels
• Functions o Sine o Cosine o Tangent • Key Features o Intercepts o intervals ➢ increasing or decreasing ➢ positive or negative o relative maximums and minimums o symmetries o periodicity o midline o amplitude • Technology (graphing complicated functions)	• Graph • Show key features • Use technology	3 4 3

Algebra II
Unit 4: Trigonometric Functions

Concepts What Students Need to Know	Skills What Students Need To Be Able To Do	Bloom's Taxonomy Levels
• Unit circle • Extension of trig functions to all Reals • Radian measure; counterclockwise motion about unit circle	• Explain (extension)	4
• Periodic phenomena given specified o Period o Amplitude o Midline	• Model periodic phenomena	4

Algebra II
Unit 4: Trigonometric Functions

Essential Questions
When does a function best model a situation?
Corresponding Big Ideas
Trigonometric functions are useful for modeling periodic phenomena.
Standardized Assessment Correlations (State, College and Career)
CollegeBoard PSAT and SAT

Unit Vocabulary
Radian, central angle, intercepted arc, length, unit circle, special right triangles, degree, sine, cosine, tangent, ratio, quadrant, reflect, congruent, reference angle, amplitude, frequency, midline, trigonometric function, periodic function

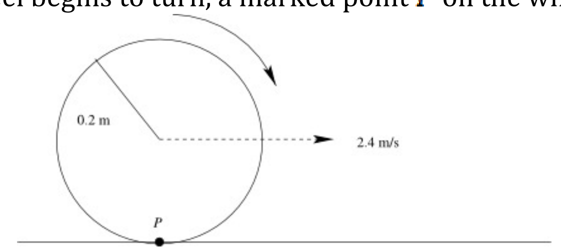
Unit Assessments The items developed for this section can be used during the course of instruction when deemed appropriate by the teacher.
Section quizzes, End-of-unit test

Learning Activities

Topic	Section in Text	CCSS
Understanding periodic data	13.1	CC.9-12.F.TF.5
Radian Measure & Angles in the Unit Circle	13.2 – 13.3	CC.9-12.F.TF.2 CC.9-12.F.TF.3
Sine, Cosine, and Tangent Functions	13.4 – 13.6	CC.9-12.F.IF.7e
Translating Sine and Cosine Functions	13.7	CC.9-12.F.IF.7e CC.9-12.F.BF.3

Application

A wheel of radius 0.2 meters begins to move along a flat surface so that the center of the wheel moves forward at a constant speed of 2.4 meters per second. At the moment the wheel begins to turn, a marked point P on the wheel is touching the flat surface.



a. Write an algebraic expression for the function y that gives the height (in meters) of the point P, measured from the flat surface, as a function of t, the number of seconds after the wheel begins moving.

b. Sketch a graph of the function y for $t > 0$. What do you notice about the graph? Explain your observations in terms of the real-world context given in this problem.

Algebra II
Unit 5: Exponential and Logarithmic Functions

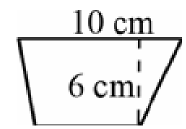
Pacing: 5 weeks + 1 week for reteaching/enrichment

Mathematical Practices
Mathematical Practices #1 and #3 describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. *Practices in bold are to be emphasized in the unit.* 1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the reasoning of others. **4. Model with mathematics.** **5. Use appropriate tools strategically.** 6. Attend to precision. **7. Look for and make use of structure.** **8. Look for and express regularity in repeated reasoning.**
Standards Overview
Analyze functions using different representations. **Construct and compare linear, quadratic, and exponential models and solve problems.**

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	
CC.9-12.F.IF.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^{t}$, $y = (0.97)^{t}$, $y = (1.01)^{12t}$, $y = (1.2)^{(t/10)}$, and classify them as representing exponential growth or decay	
CC.9-12.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.*	Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret there meaning in terms of a context.
CC.9-12.A.SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^{n}$ as the product of P and a factor not depending on P	
CC.9-12.A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.*	Example: ● In February, the Bezanson family starts saving for a trip to Australia in September. The Bezanson's expect their vacation to cost $5375. They start with $525. Each month they plan to deposit 20% more than the previous month. Will they have enough money for their trip?

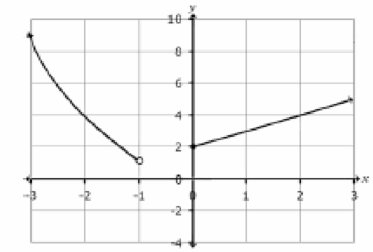
Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. Examples: • Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.  Lava coming from the eruption of a volcano follows a parabolic path. The height *h* in feet of a piece of lava *t* seconds after it is ejected from the volcano is given by $h(t)= -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet?
CC.9-12.F.BF.1 Write a function that describes a relationship between two quantities.*	Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. Examples: • You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference
CC.9-12.F.BF.1b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model	

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

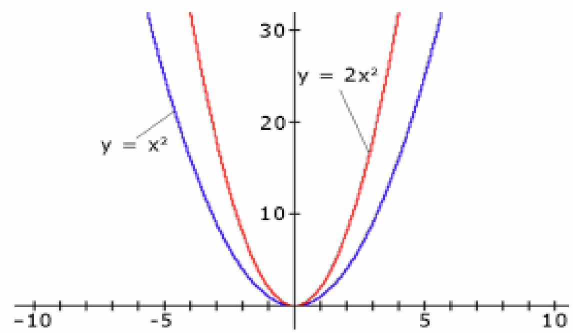
Priority and Supporting **CCSS**	**Explanations and Examples***
	between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. ● The radius of a circular oil slick after *t* hours is given in feet by $r = 10t^2 - 0.5t$, for $0 \leq t \leq 10$. Find the area of the oil slick as a function of time.
CC.9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

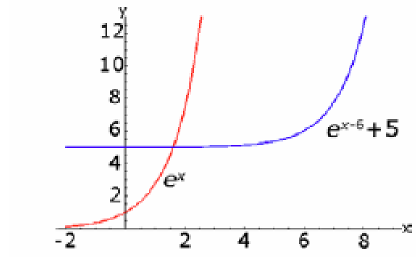
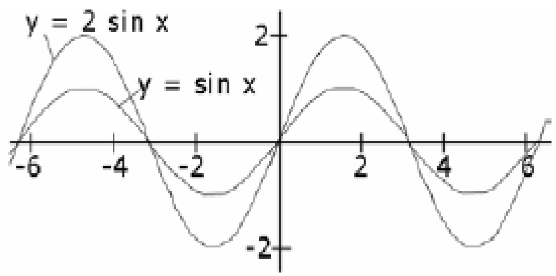
Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude	Examples: • Describe key characteristics of the graph of $f(x) = \lvert x-3 \rvert + 5$. • Sketch the graph and identify the key characteristics of the function described below. $F(x) = \begin{cases} x+2 \text{ for } x \geq 0 \\ -x^2 \text{ for } x < -1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x - 1$. Describe its domain, range, intercepts, and asymptotes. • Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?
CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
CC.9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and	Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. Examples: • A rocket is launched from 180 feet above the ground at time t = 0. The function that models this situation is given by h = – 16t2 + 96t + 180, where t is measured in seconds and h is height above the ground measured in feet.

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
periodicity.*	o What is a reasonable domain restriction for *t* in this context? o Determine the height of the rocket two seconds after it was launched. o Determine the maximum height obtained by the rocket. o Determine the time when the rocket is 100 feet above the ground. o Determine the time at which the rocket hits the ground. o How would you refine your answer to the first question based on your response to the second and fifth questions? ● Compare the graphs of $y = 3x^2$ and $y = 3x^3$. ● Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$. ● Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease. ● It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions  • Describe effect of varying the parameters *a, h,* and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$. • Compare the shape and position of the graphs of $f(x) = e^x$ to $g(x) = e^{x-6} + 5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions

Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
	 • Describe the effect of varying the parameters *a, h,* and *k* on the shape and position of the graph f(*x*) = $ab^{(x+h)}$ + k., orally or in written format. What effect do values between 0 and 1 have? What effect do negative values have? • Compare the shape and position of the graphs of *y* = sin *x* to *y* = 2 sin *x.* 

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
CC.9-12.A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions Example: ● Given the following equations determine the *x* value that results in an equal output for both functions. $f(x) = 3x - 2$ $g(x) = (x + 3)^2 - 1$
CC.9-12.F.LE.4 For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms. Example: ● Solve 200 *e*0.04*t* = 450 for *t*. Solution: We first isolate the exponential part by dividing both sides of the equation by 200. $e^{0.04t} = 2.25$ Now we take the natural logarithm of both sides. $\ln e^{0.04t} = \ln 2.25$ The left hand side simplifies to 0.04*t*, by logarithmic identity 1. $0.04t = \ln 2.25$ Lastly, divide both sides by 0.04

Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

Priority and Supporting **CCSS**	**Explanations and Examples***
	$t = ln\ (2.25) / 0.04$ $t \approx 20.3$
CC.9-12.F.BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to solve problems involving logarithms and exponents. Example: ● Find the inverse of $f(x) = 3(10)^{2x}$.

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 5: Exponential and Logarithmic Functions

Concepts What Students Need to Know	Skills What Students Need To Be Able To Do	Bloom's Taxonomy Levels
● Equivalent forms of expression	● Write a function to model	4
● Properties of exponents	● Use properties of exponents	3
● Exponential growth or decay	● Interpret / Classify growth or decay	4
● Functions (expressed symbolically)	● Graph	3
o Exponential	● Show key features – intercepts & end behavior	4
o Logarithmic		
● Key Features	● Use technology	3
o Intercepts		
o intervals		
➢ increasing or decreasing		
➢ positive or negative		
o end behavior / asymptotes		
● Technology (graphing complicated functions)		
● Exponential / logarithmic form	● Express as logarithm	3
● Logarithm	● Evaluate logarithm	2

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Algebra II
Unit 5: Exponential and Logarithmic Functions

Essential Questions
When does a function best model a situation?
Corresponding Big Ideas
Lines, exponential functions, and parabolas each describe a specific pattern of change.
Standardized Assessment Correlations (State, College and Career)
CollegeBoard PSAT and SAT

Unit Vocabulary
Exponential Function, growth/decay factor, growth/decay rate, geometric series, asymptote/end behavior, intercepts, “e” as an irrational number, compound interest applications; continuous, yearly, monthly, quarterly, biweekly, semiannually, logarithm, common logarithm, logarithmic function, domain and range, product property of logs, quotient property of logs, power property of logs, exponential and logarithmic equations, change of base formula, natural logarithms, inverse functions

Unit Assessments **The items developed for this section can be used during the course of instruction when deemed appropriate by the teacher.**
Section quizzes End-of-unit test

Algebra II
Unit 5: Exponential and Logarithmic Functions

Learning Activities

Topic	Section in Text	CCSS
Derive the exponential function as a formula for the sum of a finite geometric series (when common ratio is 1)	PH Algebra 2 8.1 Pre-Calc 5.3	A.SSE.4
Interpret exponential functions • Find the y-intercept • Find the horizontal asymptote • Find zero – if it exists • Identify growth/decay rate • Identify growth/decay factor • Irrational number "e"	PH Algebra 2 8.1 8.2 Pre-Calc 5.3	F.IF.8 F.IF.8b
Interpret the parts of the exponential function in context • Growth/decay rate • Growth/decay factor • Initial/starting amount	PH Algebra 2 8.1 8.2 Pre-Calc 5.3	A.SSE.1 A.SSE.1b
Create/build an exponential function	PH Algebra 2	A.CED.1

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

• Given initial amount and growth/decay factor • Given initial amount and growth/decay rate • Given two points • Increasing vs decreasing function • Use the created function to answer other related questions	8.1 8.2 Pre-Calc 5.3 5.7 5.8 5.9	F.BF.1 F.BF.1b F.IF.4
Graph exponential function and show key features • Y-intercept • Horizontal asymptote • Key points: f(1) and f(-1) • Use knowledge of transformations to graph	PH Algebra 2 8.1 8.2 Pre-Calc 5.3	F.IF.7 F.IF.7e F.BF.3
Understand the logarithmic function as an inverse of the exponential function • Know the definition of a logarithmic function • Evaluate a logarithm – with common bases	PH Algebra 2 8.3	F.BF.5(+)
Graph logarithmic function and show	PH Algebra 2	F.BF.3

Adapted from The Leadership and Learning Center “Rigorous Curriculum Design” model.
*Adapted from the Arizona Academic Content Standards.

key features • Zero (x-intercept) • Vertical asymptote • Key points: f(x) = 1 and f(x) = -1 • Use knowledge of transformations to graph	8.3 Pre-Calc 5.4	F.IF.7 F.IF.7eS
Solve exponential and logarithmic equations • Use definition of log to transform to exponential and solve • Use properties of logs to express equation as single log and then solve • Use Change of Base formula to solve log equations without a base of ten • Use technology to solve exponential and log equations	PH Algebra 2 8.4 8.5 8.6 Activity Lab p. 476 Pre-Calc 5.4 5.5 5.6 5.7 5.8 5.9	A.REI.11 F.LE.4 F.BF.1 F.BF.5(+) A.CED.1 A.CED.2

Adapted from The Leadership and Learning Center "Rigorous Curriculum Design" model.
*Adapted from the Arizona Academic Content Standards.

Application

1. A pie is taken out of a $350°F$ oven and set on the counter to cool. The pie's temperature decreases at a rate of 3% each minute.

   a. Does this model exponential growth or decay?
   b. What is the temperature of the pie when it is first taken out of the oven?
   c. What is the growth/decay rate?
   d. What is the growth/decay factor?
   e. Using the form $f(x) = a \cdot b^x$, write an exponential function modeling the pie's temperature $T(x)$ as a function of time, x, in minutes.
   f. Using your function, predict the pie's temperature 30 minutes after it is taken out of the oven. Round your answer to the nearest whole degree.

2. Jonas purchased a new car for $15,000. Each year the value of the car depreciates by 30% of its value the previous year. In how many years will the car be worth $500?